Improved Predictions of Trailing-edge Noise using Rapid-distortion Theory and CFD Data

Mohammed Z. Afsar[1]

[1]Department of Mechanical & Aerospace Engineering, University of Strathclyde,
75 Montrose Street, Glasgow G1 1XJ, UK.

ABSTRACT

We show the effect of correcting the acoustic spectrum formula for a high speed jet interacting with a trailing edge of a semi-infinite flat plate derived by Goldstein *et al*., (2017, J Fluid Mech. vol. 824, p. 477) using the Rapid-distortion theory (RDT). The correction involves using the next order term for the amplitude function in the WKBJ approximation for the scattered pressure. The high frequency roll-off of the acoustic spectrum predictions that we obtain using this correction are closer to experiment for low speed jets. In the talk we will discuss the role of using CFD data in the model.

1. Introduction

Rapid-distortion theory uses linear analysis to study the interaction of turbulence with solid surfaces. For example, jet-surface interaction noise in Fig. 1 occur when surfaces play a direct role in the generation of sound and/or its propagation.

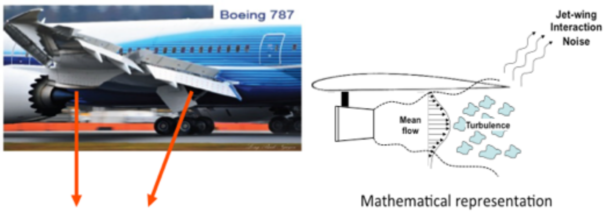


Fig. 1 Modeling edge noise using RDT.

RDT applies when the turbulence intensity is taken as a global small parameter everywhere in the flow; i.e. $\alpha \equiv |\boldsymbol{u}|/U_J \ll O(1)$ where α is the turbulence intensity and $|\boldsymbol{u}|$ is the magnitude of the local rms turbulence velocity and U_J is the nozzle exit velocity. It also implies that the length (or time) scale over which the interaction takes place is short compared to the length (or time) scale over which the turbulent eddies evolve. Hence, when interpreted asymptotically, these assumptions imply, that it is possible to identify a distance that is very (infinitely) large on the scale of the interaction, but still small on the scale over which the turbulent eddies evolve. These assumptions imply that the interacting flow can be taken as inviscid and non-heat-conducting. It is, therefore, governed by the linearized Euler equations linearized about an arbitrary base flow. Hence, RDT can be thought of as the lowest order perturbation of the Navier-Stokes equations in the small turbulence intensity, α, in the vicinity of the trailing edge. The interaction problem is, therefore, linear and inviscid and the mean flow near the trailing edge is nearly transversely sheared.

Using Green's theorem, it was shown in GLA [1] that the Euler equations possess basic solutions for the inviscid pressure perturbation, $p' = p - \bar{p}$ & mass flux, $\boldsymbol{u} \equiv \rho \boldsymbol{v}'$, (where $\boldsymbol{v}'$ denotes the velocity perturbation):

$$p'(\boldsymbol{x},t) = \frac{D_0^3}{Dt^3}\int_{-T}^{T}\int_{V(\boldsymbol{y})} g(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)\tilde{\omega}_c(\tau - y_1/U(\boldsymbol{y}_T),\boldsymbol{y}_T)\,d\boldsymbol{y}d\tau \tag{1}$$

$$\rho v'_{\perp} = -\frac{\partial U/\partial x_i}{|\nabla U|}\int_{-T}^{T}\int_{V(\boldsymbol{y})} g_i(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)\tilde{\omega}_c(\tau - y_1/U(\boldsymbol{y}_T),\boldsymbol{y}_T)\,d\boldsymbol{y}d\tau \tag{2}$$

respectively where T denotes a very large, but finite, time interval. The solid surfaces $S(\boldsymbol{y})$ bound volume $V(\boldsymbol{y})$ in formulae (1) & (2) can be finite, semi-infinite or infinite in the streamwise direction but its generators must be parallel to the level curves of the mean velocity field. The Green's function, $g_i(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)$, is linearly related to $g(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)$, which is determined for incoming wave behavior as $|\boldsymbol{y}| \to \infty$ and appropriate boundary conditions on $S(\boldsymbol{y})$. The convective derivative in (1) is $D/Dt = \partial/\partial t + U\partial/\partial x_1$ and mean flow gradient in (2) is given by $\partial U(\boldsymbol{y}_T)/\partial y_i$ where $\boldsymbol{y}_T = (y_2, y_3)$.

2. Correction to the GLA [1] Result

Since $\tilde{\omega}_c$ satisfies $D_0\tilde{\omega}_c/D\tau \equiv 0$ by definition, for an arbitrary selection of its arguments, it can be used to specify the upstream boundary condition (i.e. the input) within a boundary value problem (BVP) that seeks to determine acoustic spectrum as its output. GLA showed that the output in this case, i.e. acoustic spectrum of the scattered pressure, $2\pi I_\omega(\boldsymbol{x}) \equiv \int_{-\infty}^{\infty} \widetilde{p^s(\boldsymbol{x},t)p^s(\boldsymbol{x},t+\tau)}\,d\tau$, where $\tilde{\bullet}$ denotes Favre average, for a planar jet with streamwise mean flow $U = U(y_2)$ interacting with trailing edge is given by:

$$I_\omega(\boldsymbol{x}) \sim \int_0^\infty\int_0^\infty D(y_2,\tilde{y}_2;\theta)Q(y_2;\omega,\theta)Q^*(\tilde{y}_2;\omega,\theta)S(y_2,\tilde{y}_2;k_3^{(s)},\omega)\,dy_2\,d\tilde{y}_2 \tag{3}$$

where $D(y_2,\tilde{y}_2;\theta)$ is a "directivity factor" determined by application of the Wiener-Hopf technique and $S(y_2,\tilde{y}_2;k_3^{(s)},\omega)$ is the space-time Fourier transform of the auto-covariance of $\tilde{\omega}_c$. GLA [1] show that this latter quantity is directly related to the auto-covariance of the transverse velocity correlation R_{22} via Eqs. (6.4), (6.6),

Corresponding author: Mohammed Afsar
E-mail address: mohammed.afsar@strath.ac.uk

(6.8), (6.9) & (6.13). They use this direct algebraic correspondence to R_{22} to determine its structure (since accurate models of R_{22} can be constructed; see Afsar *et al.* 2017 [2]).

The remaining term in (3), $Q(y_2;\omega,\theta)$, is determined by using the high frequency WKBJ approximation to the homogeneous Rayleigh equation in the form, $LP=0$ where P represents the appropriate outgoing wave solution. GLA constructed this using the first term of asymptotic series in inverse wavenumber. They found that $Q(y_2;\omega,\theta)$ is identically equal to unity at low frequencies, is bounded by $O(e^{-k_\infty y_2})$ behavior at large y_2 and is transcendentally small (i.e. $o(1)$) at $y_2=O(1)$ when the far-field wavenumber gets large as $k_\infty=\omega/c_\infty\to\infty$.

However, the $O(1/k_\infty)$ term of the amplitude function in the WKBJ ansatz enters the lowest order expansion for the scattered pressure when the hydrodynamic wave number limit is taken, $k_1=\omega/U(y_2)$. (This latter limit is required by the streamwise Fourier transform of Eqs. 1 & 2). As opposed to GLA (their Eq. 7.9), in this case the homogeneous solution to the Rayleigh equation takes the form of $P_{>}\to i\chi'\int_0^{y_2} q^{1/2}(y;\hat{k}_1,\hat{k}_3)dy \Big/ (k_\infty q^{3/4})$ (where the χ' & q terms are proportional the jet mean flow field and $(\hat{k}_1,\hat{k}_3)$ are non-dimensional wavenumbers in the streamwise and spanwise directions that are different to k_1 and are determined by the method of stationary phase since $|\boldsymbol{x}|\to\infty$; these terms shall be defined in the accompanying talk).

3. Results and Discussion

In the presentation, we show the effect of using the correction to the GLA formula (Eq. 7.18) for the specific experiment shown in Fig. 2. $(x_d,y_d)/D_J$ is the location of the plate trailing edge relative to the nozzle exit plane. In general we find that the predictions have greater roll-off at high frequencies for the lowest acoustic Mach number of the experiments in Fig. 2, [3]. In Fig. 3 we show the 90º acoustic spectrum with the GLA result together with the correction discussed above.

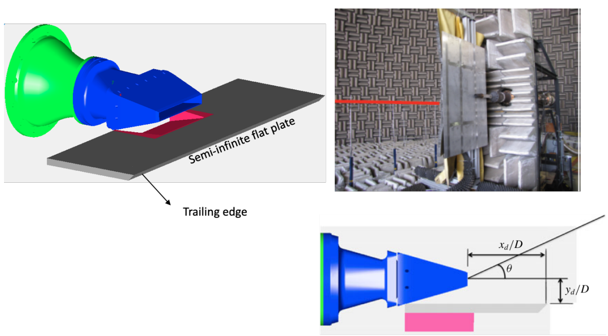
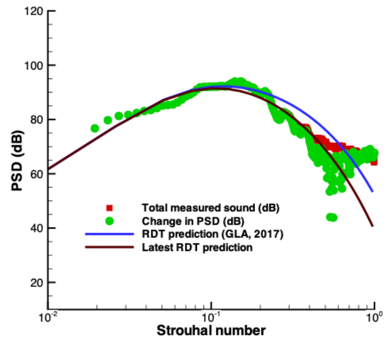


Fig. 2 Jet-plate interaction experiment [3].

(a).



(b).

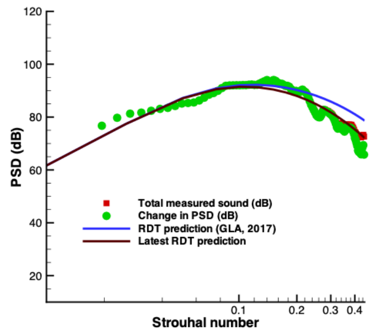


Fig. 3 Power spectral density (PSD) of the far-field pressure fluctuations at 100 equivalent diameters from nozzle exit (lossless in dB scale referenced to 20μPa) as a function of Strouhal number, for Ma=0.5. Predicted (solid line): measured data below the plate at $\psi=-90^\circ$. (Total noise: red; difference between the total noise and noise measured in the free jet: green). Plate trailing edge at $(x_d,y_d)/D=(5.7,0.98)$. (a)$\theta=90^\circ$ (b) close-up.

4. Concluding Remarks and Use of CFD Data

Our results in this paper broadly show how Rapid distortion theory can be used to model the problem of sound radiated by a turbulent jet interacting with the trailing edge of a semi-infinite flat plate. While this canonical problem was considered in Goldstein, Leib & Afsar (2017, J Fluid Mech. vol. 824, p. 477), we have analyzed the effect of using a generalized solution to the WKBJ problem in determining the appropriate formula for the acoustic spectrum. Our results in Fig. 3 show improved high frequency prediction. The use of CFD data can be used to determine the jet mean flow and will be discussed in the presentation.

References

[1] M. E. Goldstein, S. J. Leib, M. Z. Afsar, *J. Fluid Mech.*, 824, (2017), 477－512.

[2] M. Z. Afsar, S. J. Leib, R. E. Bozak, *J. Sound & Vib.*, 386, (2017), 177-207.

[3] J. E. Bridges, C. Brown, R. E. Bozak, (2014), *AIAA Paper 2014-2904.*